CALCULUS HONORS SUMMER PACKET

– You will have a quiz over this the first full week of school. You are expected to know the information included in the packet.

DOMAIN – A REVIEW

DENOMINATORS

When checking the domain of a function, the first place to look is in the denominators of fractions. The denominator can never equal 0 so any number that would make the denominator equal 0 must be excluded from the domain.

$$f(x)=\frac{x^2+11x-42}{x^2+5x-24}=\frac{x^2+11x-42}{(x-3)(x+8)} \qquad x\neq 3,-8$$

It doesn't matter if the numerator will factor or not. Only the denominator determines the domain.

$f(x)=\dfrac{x^2-3x+4}{x^2+2x+3}$ The denominator won't factor, so use the quadratic formula.

That gives you $\dfrac{-2\pm\sqrt{-8}}{2}$. Since the solution is imaginary, there are no numbers that need to be excluded from the domain. The domain is all real numbers.

$f(x)=\dfrac{3x}{x^2+3x-5}$ Again the denominator won't factor but this time the quadratic formula gives you $\dfrac{-3\pm\sqrt{29}}{2}$. Since these are real numbers, they must be excluded from the domain. $x\neq\dfrac{-3\pm\sqrt{29}}{2}$

RADICALS

The term inside a square root (or any even numbered root) can never be negative so the term inside must be ≥ 0.

$$f(x)=\sqrt{11x-2} \qquad 11x-2\geq 0 \qquad x\geq\frac{2}{11}$$

$f(x) = \sqrt[4]{5x-15}$ $5x-15 \geq 0$ $x \geq 3$

$f(x) = \sqrt{x^2+4}$ Since the term inside the radical is always positive, the domain is all real numbers.

$f(x) = \sqrt{x^2+8x-20}$ $x^2+8x-20 \geq 0$ $(x+10)(x-2) \geq 0$

This does not mean that $x+10 \geq 0$ and $x-2 \geq 0$.

Think number line ____+__________-____________+________

-10 2

Numbers less than -10 and numbers greater than 2 will make the quadratic positive so the domain is x ≤ -10 or x ≥ 2.

The term inside a cube root (or any odd numbered root) can be positive, negative, or zero so there are no restrictions on the domain.

$f(x) = \sqrt[3]{3x+7}$ The domain is all real numbers.

However, if the cube root is in a denominator, the term inside cannot equal zero.

$f(x) = \dfrac{2x}{\sqrt[3]{3x+7}}$ $3x+7 \neq 0$ $x \neq -\dfrac{7}{3}$

Likewise if a square root is in a denominator, the term inside must be greater than 0. It cannot be negative or equal zero.

$f(x) = \dfrac{x+1}{\sqrt{2x+3}}$ $2x+3 > 0$ $x > -\dfrac{3}{2}$

$f(x) = \dfrac{(8x+9)^{3/2}}{(2x+1)^{2/3}}$ Fractional exponents are confusing for many. If it is a problem for you, change to radical form.

$$f(x)=\frac{\sqrt[2]{(8x+9)^3}}{\sqrt[3]{(2x+1)^2}}$$

The exponents inside don't matter. It is the roots that are important. The numerator is a square root and the denominator is a cube root.

$8x+9\geq 0 \qquad x\geq -\frac{9}{8}$ AND $2x+1\neq 0 \qquad x\neq -\frac{1}{2}$

Both are important for the domain.

LOGS AND NATURAL LOGS

Logs and natural logs only exist for positive numbers, so the log term must be greater than 0.

$f(x)=\log(4x-20) \qquad 4x-20>0 \qquad x>5$

$f(x)=\log(2-x) \qquad 2-x>0 \qquad x<2$

$f(x)=\ln(1-\ln x)$ This is a tricky one. First of all deal with the ln x inside the parentheses. x > 0 for that part. However, there is another ln. The term inside the ( ) also must be > 0.

So 1 – lnx > 0 -ln x > -1 ln x < 1 so x < e

The domain is (0, e)

TO SUMMARIZE

First think DENOMINATOR

Then look for RADICALS WITH EVEN ROOTS

Then look for LOGS or NATURAL LOGS

DOMAIN PRACTICE PROBLEMS

YOU SHOULD TRY THESE WITHOUT USING A CALCULATOR

1. $f(x)=\sqrt{3x}$

2. $f(x)=\dfrac{x-3}{x^2-16}$

3. $f(x)=\dfrac{\ln(2x-8)}{\ln(x+5)}$

4. $f(x)=\dfrac{22e^{3x}-5e^x+2}{e^x}$

5. $f(x)=\sqrt{x^2-20x-44}$

6. $f(x)=\dfrac{3x^3-5x^2+11x}{x^2+3}$

7. $f(x)=\dfrac{x^2-5x-6}{x^2+3x}$

8. $f(x)=\dfrac{(4x-1)^{1/2}}{(3x-10)^{1/3}}$

9. $f(x)=\dfrac{(3x-10)^{1/3}}{(4x-1)^{1/2}}$

10. $f(x)=\dfrac{x-3}{\sqrt{x^2+18x+80}}$

ANSWERS TO DOMAIN PRACTICE PROBLEMS

1. $x \geq 0$

2. $x \neq \pm 4$

3. From the numerator, x > 4, from the denominator x > -5. The domain is the most restrictive one, so the domain is x > 4.

4. All real numbers (e^x can never equal 0)

5. x ≤ -2 or x ≥ 22

6. All real numbers ($x^2 + 3$ can never equal 0)

7. $x \neq 0 \quad x \neq -3$

8. $x \geq \dfrac{1}{4}$ $\quad x \neq \dfrac{10}{3}$

9. $x > \dfrac{1}{4}$

10. x < -10 or x > -8

MORE DOMAIN PRACTICE PROBLEMS
YOU SHOULD TRY THESE WITHOUT USING A CALCULATOR

1. $f(x) = \sqrt{8x^2 - 10x - 3}$

2. $f(x) = \dfrac{3x^2 - 5x + 3}{2x^3 + 5x^2 - 18x - 45}$

3. $f(x) = \dfrac{\sqrt{8x+9}}{\sqrt{3x+7}}$

4. $f(x) = \dfrac{e^{2x} - e^x - 12}{e^{2x} - 2e^x - 8}$

5. $f(x) = \dfrac{e^{2x} + 2e^x + 1}{e^{2x} - 7e^x + 10}$

6. $f(x) = \dfrac{\sqrt[3]{x^2 - 8x + 12}}{e^{2x} + 10e^x + 21}$

7. $f(x) = \ln(3 - x)$

8. $f(x) = \dfrac{1}{\ln(x^2 + 9)}$

9. $f(x)=\frac{1}{x}-\frac{2}{x^2-9}+\frac{3}{x^2-25}$

10. $f(x)=\sqrt{x-9}+\sqrt{x+3}-\sqrt{3x-16}$

11. $f(x)=\sqrt[5]{x^2+12x+32}$

ANSWERS TO MORE DOMAIN PRACTICE PROBLEMS

1. Factor and use a number line $\left(-\infty,-\frac{1}{4}\right]\left[\frac{3}{2},\infty\right)$

2. Numerator does not matter. Factor the denominator by grouping. Set not equal to 0.

   $x \neq \pm 3 \quad x \neq -\frac{5}{2}$

3. Evaluate each square root individually. Choose the more restrictive domain.

   Numerator $x \geq -\frac{9}{8}$ Denominator $x > -\frac{7}{3}$ The numerator is more restrictive so the domain of the entire fraction is $x \geq -\frac{9}{8}$

4. Numerator does not matter. Factor the denominator, set not equal to 0.

   $x \neq \ln(4)$ $\ln(-2)$ is imaginary, does not affect domain

5. Numerator does not matter. Factor the denominator, set not equal to 0.

   $x \neq \ln(2)$ $x \neq \ln(5)$

6. Numerator does not matter since it is a cube root. Factor the denominator, set not equal to 0. $x=\ln(-3)$ and $x=\ln(-7)$ are both imaginary, so the domain is

   $(-\infty,\infty)$

7. Natural logs must be > 0. 3 – x > 0 x < 3

8. The denominator is always > 0. $(-\infty,\infty)$

9. There is no need to combine these fractions. Just look at each denominator individually.

   $x \neq 0,-3,3,-5,5$

10. You have to look at each one of these individually then take the most restrictive domain.

From the first one x ≥ 9, from the second one x ≥ -3, from the third one x ≥ $-\frac{16}{3}$.

The most restrictive domain is the first one, so the domain of this function is x ≥ 9

11. Odd numbered root. Domain is $(-\infty,\infty)$

FACTORING

One of the most important topics to master, if you wish to be successful in Calculus or any other upper level math course, is factoring. By this point, you should have been exposed to many different types of factoring.

Factoring types:

The first thing to always look for is a **common factor**. Many times finding the common factor is the key to being able to successfully factor an expression.

Example: $2x^5 - 18x^4 + 42x^2 = 2x^2(x^3 - 9x^2 + 21)$

$x^2 + bx + c$ Find two numbers that multiply to make c and add to make a.

$x^2 + 10x + 21 = (x+7)(x+3)$

Signs are important. $x^2 + 19x - 42 = (x+21)(x-2)$

$ax^2 + bx + c$ The "ac" method. Find two numbers that multiply to make ac and add to make b.

$4x^2 - 8x - 21$ ac = -84 Multiply to make -84 and add to make -8. The numbers are -14 and 6.

Rewrite as $4x^2 - 14x + 6x - 21$

$2x(2x-7) + 3(2x-7) = (2x+3)(2x-7)$

Grouping method (only applies to polynomials with an even number of terms).

$10x^5 + 14x^3 + 55x^2 + 77 = 2x^3(5x^2+7) + 11(5x^2+7) = (2x^3+11)(5x^2+7)$

Differences of squares $a^2 - b^2 = (a-b)(a+b)$ $81x^2 - 49y^2 = (9x-7y)(9x+7y)$

Sums of squares will not factor.

Differences of cubes $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ $8x^3 - 125 = (2x-5)(4x^2+10x+25)$

The second parenthesis will never factor.

Sums of cubes $a^3 + b^3 = (a+b)(a^2-ab+b^2)$ $27x^3 + 343y^3 = (3x+7y)(9x^2-21xy+49y^2)$

The second parenthesis will never factor.

Quadratic types $x^4 + 13x^2 + 40$ Even though this is not a quadratic expression, it can still be treated similar to one. Two numbers that multiply to make 40 and add to make 13. 5 and 8. Factoring: $(x^2 + 5)(x^2 + 8)$ Any trinomial that fits the pattern where the largest exponent is twice as big as the other exponent and the third term has no variable fits the pattern.

$24x^6 – 14x^3 – 3$ ac = -72 Two numbers that multiply to make -72 and add to make -14
-18 and 4
Answer: $(4x^3 – 3)(6x^3 + 1)$

Many problems involve combinations of factoring types.

$640x^7 – 360x^5 + 80x^4 – 45x^2$ Common factor is $5x^2$
$5x^2 (128x^5 – 72x^3 + 16x^2 – 9)$ Now use grouping
$5x^2[8x^3 (16x^2 – 9) + 1(16x^2 – 9)]$
$5x^2 (8x^3 +1)(16x^2 – 9)$ First parenthesis is sum of cubes, second is difference of squares.
$5x^2 (2x + 1)(4x^2 – 2x + 1)(4x – 4)(4x + 3)$

Factor each expression below as completely as possible. If the expression cannot be factored, write PRIME.

1. $x^2 – 19x – 90$

2. $x^2 – x – 2$

3. $x^2 – 22x + 120$

4. $49x^5 – 36x^3$

5. $2x^5 + 11x^3 + 10x^2 + 55$

6. $40x^2 + 23x + 3$

7. $x^2 + 60x + 900$

8. $121x^3 – 225xy^2$

9. $x^6 - 9x^3 + 8$

10. $18x^2 + 23x - 6$

11. $135x^4 + 320x$

12. $5x^2 + 11xy + 2y^2$

13. $8x^3 + 112x^2 - x - 14$

14. $x^6 - x^4 - x^2 + 1$

15. $e^{2x} - 6e^x - 55$

16. $16x^3 + 48x^2 - 9x - 27$

17. $8x^5 - 22x^3 + 12x$

18. $x^2 - 101x + 100$

Finding Common Denominators with Rational Functions

If you have a rational expression over another rational expression added or subtracted over another fraction such as $\frac{\frac{1}{4}+5}{6}$ then you need to get a common denominator with the numerator before even looking at the 6. You get a common denominator with 5 to make it 20/4, so now it becomes $\frac{\frac{1}{4}+\frac{20}{4}}{6}$. Now, the 4 moves down to the 6 and gets multiplied to become 24. The 1 is added to the 20 to get 21, so you now have $\frac{21}{24}$

Here's another example $\frac{(12x-1)-\left(\frac{1}{10}\right)\left(\frac{1}{x}\right)}{5x}$. First, multiply 1/x and 1/10, then find the common denominator in the numerator. It is 10x. Now, you need to multiply the (12x-1) with the 10x, so the numerator becomes $\frac{\frac{10x(12x-1)}{10x}-\frac{1}{10x}}{5x}$ this then simplifies to $\frac{(10x)(12x-1)-1}{5x(10x)}$. Combine like terms to get $\frac{120x^2-10x-1}{5x(10x)}$.

Let's work out some problems now. Simplify each rational expression as much as possible.

1) $\frac{(10x+3)-\left(\frac{2x}{5}\right)\left(\frac{1}{3x}\right)}{4}$

2) $\frac{\frac{1}{ln5}-\frac{3x^2}{2}}{5x}$

3) $\frac{(\frac{1}{3})-(12x^2-x-1)}{10x}$

4) $\frac{(14x-5)\left(\frac{4x}{7}\right)-\left(x^3-2x\right)(14x)}{7x^3}$

Answer KEY

1) $\frac{150x^2+43x}{60x}$

2) $\frac{2-3x^2 ln5}{10x ln5}$

3) $\frac{-36x^2+3x+4}{30x}$

4) $\frac{98x^4+56x^2+176x}{49x^3}$

*take out an x everywhere to get

$\frac{98x^3+56x+176}{49x^2}$