Name: _____ AP Calculus AB Summer Packet

Date: _____

Topics:

I. Trigonometry
II. Vectors and Polar Coordinates
III. Functions and their transformations
IV. Exponentials and Logarithms
V. Sequences and Series
VI. Limits
VII. Derivatives

This packet is due on the first day of school. It will be graded, and there will be an opportunity for you to ask questions about the material. Then you will be given a test within a few days.

I can provide links to recordings of my PreCalculus Honors class which you can use to review or learn for the first time the needed material. Email me at fletcher.williams@knoxvillecatholic.com to get them.

Assume that all of the problems included are to be completed without a calculator unless indicated otherwise.

I. Trigonometry

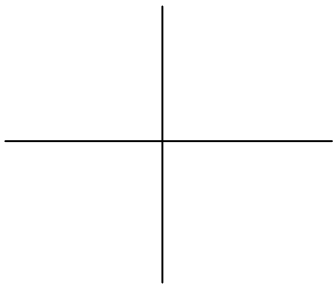
*Know your unit circle well. You should be able to calculate the following (idea: make a Quizlet and share it with others!):

- basic trig ratios, e.g. $\sin 30^\circ, \cos \pi, \sec 0, \tan 480^\circ$...

- basic inverse trig, e.g. $\arcsin(1/2), \cos^{-1} 0, \tan^{-1} 1, \operatorname{arccsc}\left(-\sqrt{2}\right)$... (be careful of restricted ranges!)

1. Graph two periods of each of the following. Label the x and y axes with key values. Under each graph, identify it as even/odd/neither symmetry, and state the domain/range.

$y = \sin x$

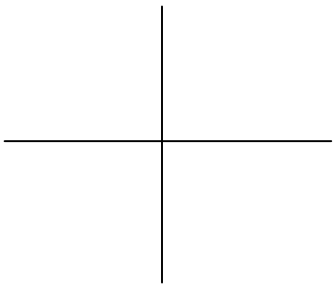


Symmetry: _____

Domain: _____

Range: _____

$y = \cos x$

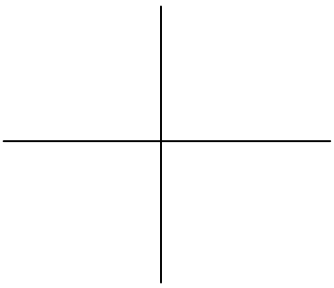


Symmetry: _____

Domain: _____

Range: _____

$y = \tan x$

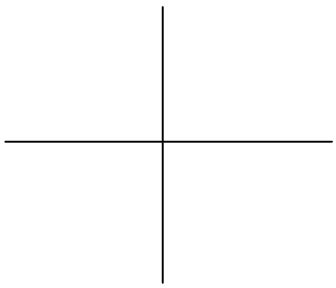


Symmetry: _____

Domain: _____

Range: _____

$y = \csc x$

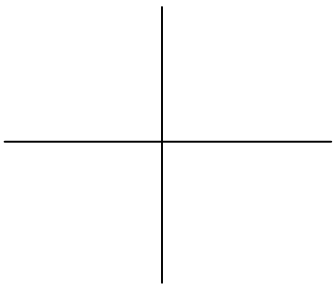


Symmetry: _____

Domain: _____

Range: _____

$y = \sec x$

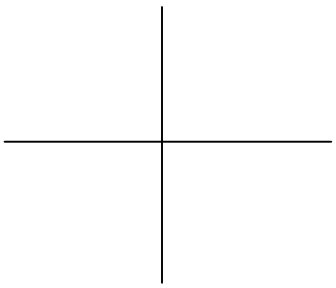


Symmetry: _____

Domain: _____

Range: _____

$y = \cot x$



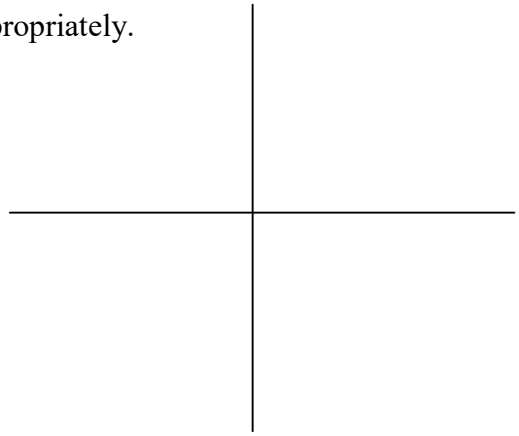
Symmetry: _____

Domain: _____

Range: _____

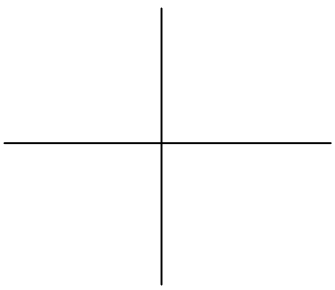
2. Write the equation of a sine function that has a max of 12, a min of 0, and a period of 10.

3. Graph one period of $y = -2\cos\left(\frac{\pi}{4}x + \pi\right) - 1$. Label appropriately.



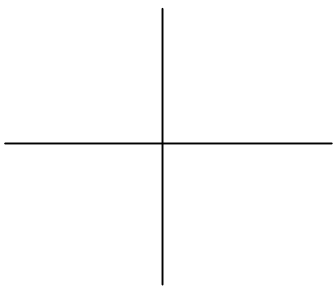
4. You should understand how the inverse trig functions are obtained from the ordinary ones. As an example, graph the following two inverse trig functions as well as the restricted-domain versions of the functions used to generate them.

$y = \sin x$

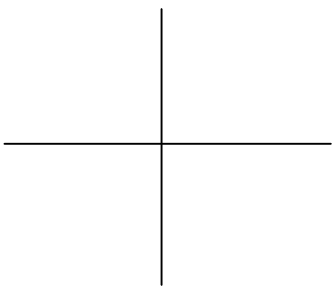


(restricted domain)

$y = \sin^{-1} x$

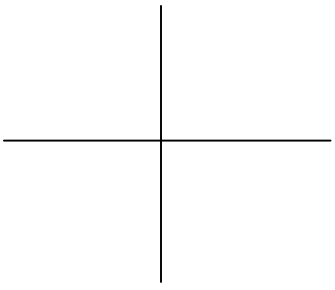


$y = \tan x$



(restricted domain)

$y = \arctan x$



5. Calculations with trig identities.
   a. Calculate $\sin 255^\circ$

   b. Simplify $10\sin 3\alpha \cos 3\alpha$ to a trig function of a single angle.

6. Calculations with inverse trig functions:
   a. Calculate $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

   b. Simplify $\sin\left(\cos^{-1} x\right)$ to an expression with no trig functions.

7. Solve the following equations on the interval $0 \le \theta < 360^\circ, 0 \le x < 2\pi$.
   a. $\cos 2x = \sin x$

   b. $\sin 2\theta = 2\sin\theta$

II. Vectors and Polar Coordinates

8. Given two vectors $\vec{A}$ and $\vec{B}$ with $\left\|\vec{A}\right\| = 3,\ \left\|\vec{B}\right\| = 8$ and that the two vectors make angles with respect to the positive x-axis of 70° and 280°, respectively. (Calculator OK).
    a. Draw a diagram of the vector $\vec{C} = \vec{B} - 2\vec{A}$, using the "tip-to-tail" triangle method.
    b. Calculate $\left\|\vec{C}\right\|$ as well as the angle that $\vec{C}$ makes with the positive x axis.
    c. Calculate $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$.
    d. Write $\vec{A}, \vec{B}$ in component form.

9. Given two vectors $\vec{v} = 5\hat{i} - 4\hat{j} + 7\hat{k},\ \vec{w} = -\hat{i} - 10\hat{j} + 3\hat{k},$ calculate:
    a. $\vec{v} + \vec{w}$
    b. $2\vec{v} - 3\vec{w}$
    c. $\vec{v} \cdot \vec{w}$
    d. $\vec{v} \times \vec{w}$
    e. Determine the angle between $\vec{v}, \vec{w}.$
    f. Determine a unit vector perpendicular to $\vec{v}, \vec{w}.$

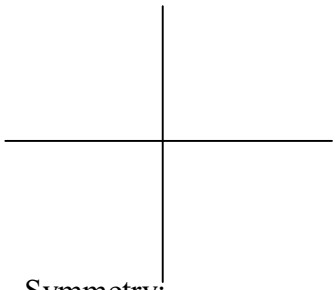
10. Convert the following equations from rectangular to polar coordinates and solve for *r*.
    a. $x + y - 2 = 0$

    b. $x^2 + y^2 + 8x = 0$

III. Functions and Their Graphs

11. Sketch the following and determine their symmetry, domain and range.

$y = x^2$

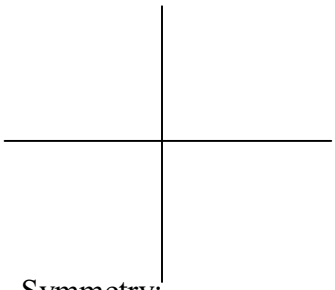


Symmetry: _____

Domain: _____

Range: _____

$y = x^3$

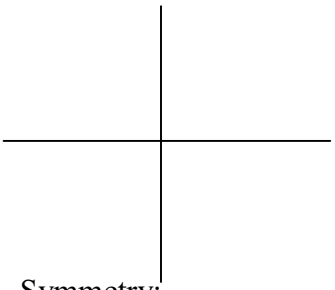


Symmetry: _____

Domain: _____

Range: _____

$y = x^4$

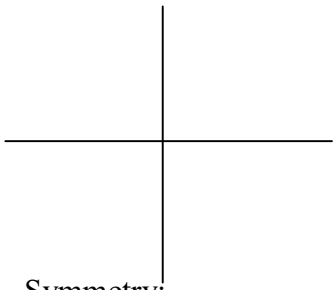


Symmetry: _____

Domain: _____

Range: _____

$y = \sqrt{x}$

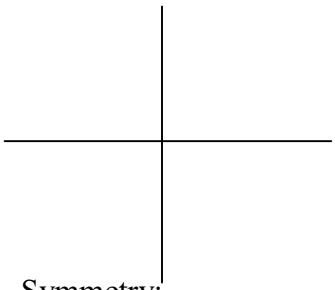


Symmetry: _____

Domain: _____

Range: _____

$y = \sqrt[3]{x}$

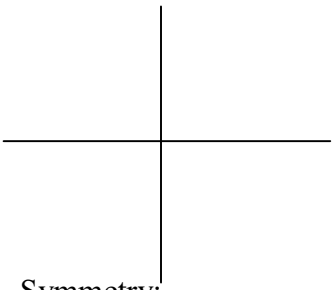


Symmetry: _____

Domain: _____

Range: _____

$y = 1/x$

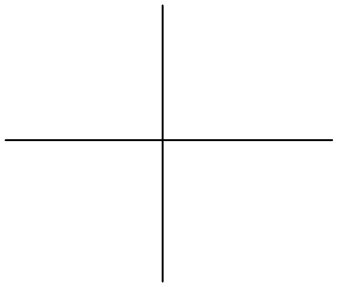


Symmetry: _____

Domain: _____

Range: _____

$y = 1/x^2$

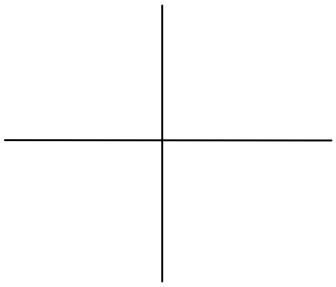


Symmetry: _____

Domain: _____

Range: _____

$y = e^x$

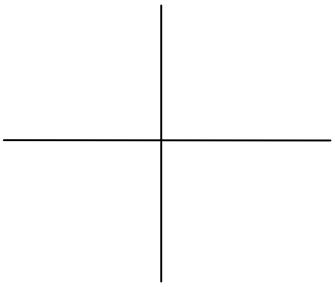


Symmetry: _____

Domain: _____

Range: _____

$y = \ln x$



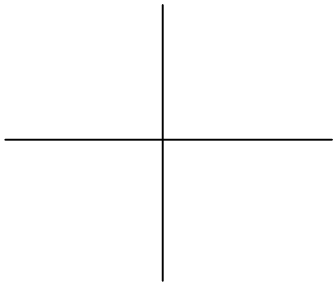
Symmetry: _____

Domain: _____

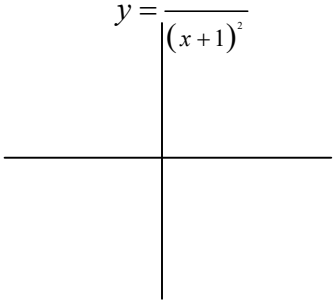
Range: _____

12. Graph the following transformed functions.

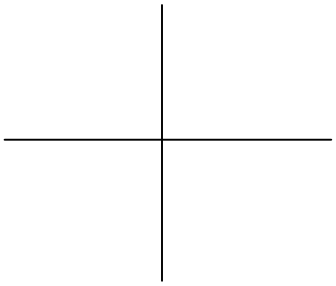
$y = -\ln(x-1)$



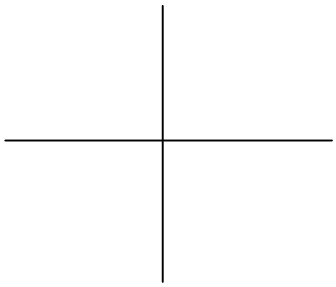
$y = \frac{1}{(x+1)^2}$



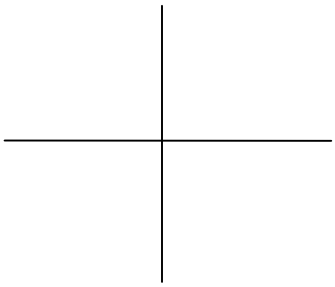
$y = 2\sqrt[3]{-x}$



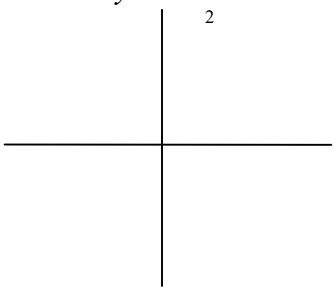
$y = \sqrt{x-1} + 2$



$y = 2^{-x}$



$y = 1 - \frac{1}{2}x^4$



IV. Exponentials and Logarithms

13. Simplify the following:

a. $5^{\log_5 3}$

b. $\log_3 27^{\sqrt{2}}$

c. $\ln \frac{1}{e^3}$

d. $\log_6\left(\frac{36}{6^{-10}}\right)$

14. Solve for x in terms of natural logs: $e^{2x-1}=3$

15. Solve for t in terms of log base 10: $10^{5t}=2$

16. Solve for x in terms of e: $\ln x^2=8$

17. Combine the following into a single logarithmic expression: $\frac{1}{3}\log_b x+\log_b y$

V. Sequences and Series

18. Write the following series in sigma notation:

a. $2+4+6+\ldots+1000$

b. $m_1r_1^{\,2}+m_2r_2^{\,2}+m_3r_3^{\,2}+\ldots+m_Nr_N^{\,2}$

c. $\frac{1}{2}-\frac{2}{3}+\frac{3}{4}-\ldots+\frac{99}{100}$

19. Write the following series in expanded form, showing at least the first three terms and the last term of the given series (if there is a last term).

d. $\sum_{n=4}^{10}(3n-2)$

e. $\sum_{j=0}^{15} \frac{(-1)^j}{j+1}$

20. Prove the following by mathematical induction:

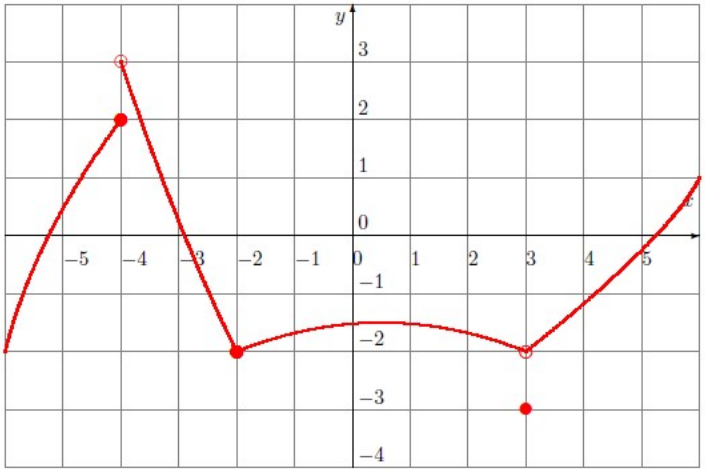
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

VI. Limits

21. Write down the definition of a limit. (That is, what is formally required for the statement $\lim_{x \to a} f(x) = \text{L}$ to be true.) Then graph a function and label L, ε, and a and then use that graph to write out a verbal explanation of the meaning of a limit.

22. Draw a graph of an arbitrary function $f(x)$ and use it to derive the limit definition of the derivative. To do so, label two points (thoughtfully), and draw a secant line through them. Then go from there…

23. Calculate the indicated limits/function values based on the graph:



a) $\lim_{x\to -4^-} f(x)$

b) $\lim_{x\to -4^+} f(x)$

c) $\lim_{x\to -4} f(x)$

d) $\lim_{x\to -2} f(x)$

e) $\lim_{x\to 3} f(x)$

f) $\lim_{x\to -2^-} f(x)$

g) $f(-4)$

h) $f(-2)$

i) $f(3)$

24. Given that $\frac{\sin 3x}{x} \le f(x) \le x^2 + 3$ on the interval [- 5, 5], determine $\lim_{x\to 0} f(x)$ and justify your answer.

VII. Derivatives

25. Know the following derivative rules by heart (make flashcards) & then compute: Derivative of a constant, Power Rule, Constant Multiple Rule, Sum Rule, Product Rule, Quotient Rule, Trig (note that $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$)

a. $v(t) = t^5 - 4t + \sqrt{t} - 3 - \frac{1}{t^2}$

b. $y = \sqrt{1 + x^2}$

c. $f(x) = \dfrac{\sin x - \cos x}{\sin x + \cos x}$

d. $f(x) = \left(2x^2 - 3\right)^4 \left(2x + 4\right)^3$

e. $g(x) = \left(\dfrac{x^2 - 1}{x^2 + 1}\right)^2$

26. Write down the equation of the line tangent to $y = \sqrt[3]{x}$ at x = - 8.